Muon anomalous magnetic moment and lepton flavor violation in MSSM

X.-J. Bi[a], Y.-p. Kuang[b], Y.-H. An

Department of Physics, Tsinghua University, Beijing 100084, P.R. China

Abstract. We give a thorough analysis of the correlation between the muon anomalous magnetic moment and the radiative lepton flavor violating (LFV) processes within the minimal supersymmetric standard model. We find that in the case when the slepton mass eigenstates are nearly degenerate, δa_μ, coming from SUSY contributions, hardly depends on the lepton flavor mixing and, thus, there is no direct relation between δa_μ and the LFV processes. On the contrary, if the first two generations' sleptons are much heavier than the third one, i.e., in the effective SUSY scenario, the two quantities are closely related. In the latter scenario, the SUSY parameter space to account for the experimental δa_μ is quite different from the case of no lepton flavor mixing. Especially, the Higgsino mass parameter μ can be either positive or negative.

1 Introduction

Recently, the Brookhaven E821 Collaboration announced their new experimental result on the muon anomalous magnetic moment, $a_\mu = (g_\mu - 2)/2$, with improved statistics [1], which is twice the precision of their 2001 result [2]. However, there are large uncertainties in the theoretical evaluation of a_μ within the standard model (SM), with the errors mainly coming from the hadronic corrections. According to the most updated calculations, the present discrepancy between the SM prediction and the measurement is

$$a_\mu^{\exp} - a_\mu^{\mathrm{SM}} = 36(11) \times 10^{-10} \ [e^+e^- - \text{based}] \ [3] \quad (1)$$

$$\text{or } 34(11) \times 10^{-10} \ [e^+e^- - \text{based}] \ [4] \quad (2)$$

$$\text{or } \ 9(10) \times 10^{-10} \ [\tau - \text{based}] \ [4] \,, \quad (3)$$

lying from about 3σ to 1σ. In the above expression, $[e^+e^-$-based] and $[\tau$-based] mean that the calculations are based on the experimental data from e^+e^- annihilation and τ decays respectively. We will take $\delta a_\mu = 34(\pm 11) \times 10^{-10}$ in our later analysis.

Since the first announcement of an existing discrepancy between theory and experiment on a_μ, there appeared a lot of works on this subject trying to explain the result in various extensions of the SM, among which the most promising new physics is the minimal supersymmetric standard model (MSSM) [5]. Although the present E821's measurement cannot provide compelling evidence in favor of new physics, it is generally expected that this

deviation will be confirmed when both the experimental and theoretical errors are reduced and this result can now be used to put constraint on the supersymmetric (SUSY) parameters.

The extensive studies show that it is easy to accommodate $\delta a_\mu \sim (10\text{–}70) \times 10^{-10}$ within the MSSM framework if the SUSY particles are as "light" as about a few hundred GeV [5]. The positive sign of the Higgsino mass parameter, μ, is strongly favored by the present value of δa_μ. Since in most of parameter space the main SUSY contribution to a_μ comes from exchanging chargino and scalar muon neutrino virtual particles, which is approximately proportional to $\mu M_2 \tan \beta$, the sign of μ is thus positive relative to M_2, the wino mass parameter, provided that SUSY helps to enhance a_μ.

In this work we will study the SUSY contributions to a_μ in the case when considering the lepton flavor mixing in the soft breaking sector. Different from the similar numerical studies of lepton flavor mixing effects on a_μ [6], we will give a thorough analysis of the correlation between the SUSY contributions to a_μ and to lepton flavor violation (LFV). We find in the case that the sleptons and sneutrinos are nearly degenerate, δa_μ has no direct relation with the LFV processes. Actually, this is the usual case which has been extensively studied in the literature [5]. However, it is most important to take into account the effects of lepton flavor mixing in the case of effective SUSY scenario [7], where δa_μ can only arise when the slepton mixing between the second and the third generations is introduced. The effective SUSY scenario (or decoupling solution) is proposed to solve the flavor changing neutral current (FCNC) and CP problems in MSSM, caused by introducing the soft SUSY breaking terms. In this scenario, the sfermions in the first and second generations are heavy

a e-mail: bixj@mail.tsinghua.edu.cn
b e-mail: ypkuang@mail.tsinghua.edu.cn

enough (heavier than about 20 TeV) to suppress their contributions to the FCNC and CP violating processes, while the sfermions of the third generation and gauginos and higgsinos are light (about a few hundred GeV) to keep the Higgs boson naturally light.

We find that in the effective SUSY scenario, the SUSY parameter space may be quite different from those without considering the slepton mixing. The sign of μ can be either positive or negative, to enhance a_μ, depending on the lepton flavor mixing angles. A small $\tan\beta$ is more favored in this case.

This paper is organized as follows. In the next section we give the analytic expressions for the SUSY contributions to a_μ and the branching ratio of LFV processes $l_i \to l_j\gamma$. In Sect. 3, we will present the numerical results and some approximate upper bound on δa_μ. Finally, we give our summary and conclusions in Sect. 4.

2 Analytic expressions

2.1 Expressions of δa_μ and $\mathrm{Br}(l_i \to l_j\gamma)$ in MSSM

Since the muon's anomalous magnetic moment and the LFV processes $l_i \to l_j\gamma$ arise from similar operators, the effective Lagrangian related to a_μ and radiative LFV process can be written in one form,

$$\mathcal{L}_{\text{eff}} = e\frac{m_i}{2}\bar{l}_j\sigma_{\alpha\beta}F^{\alpha\beta}\left(A_{\text{L}}^{ij}P_{\text{L}} + A_{\text{R}}^{ij}P_{\text{R}}\right)l_i, \qquad (4)$$

where $P_{\text{L,R}} = \frac{1}{2}(1\mp\gamma_5)$ are the chirality projection operators and i (j) denotes the initial (final) lepton flavor. The muon anomalous magnetic moment is given by

$$a_\mu = m_\mu^2\left(A_{\text{L}}^{22} + A_{\text{R}}^{22}\right), \qquad (5)$$

while the branching ratio of $l_i \to l_j\gamma$ is given by

$$\mathrm{Br}(l_i \to l_j\gamma) = \frac{\alpha_{\text{em}}}{4}m_i^5\left(|A_{\text{L}}^{ij}|^2 + |A_{\text{R}}^{ij}|^2\right)/\Gamma_i, \qquad (6)$$

with Γ_i [8] being the width of l_i.

The SUSY contribution to the form factors A_{L} and A_{R} is given by the photon-penguin diagrams via exchanging (i) chargino-sneutrino and (ii) neutralino-slepton, as shown in Fig. 1. The analytic expressions for δa_μ [9] from the neutralino and chargino exchange are

$$\delta a_\mu^{(n)} = -\frac{1}{32\pi^2}\frac{e^2}{\cos^2\theta_{\text{W}}}\frac{m_\mu^2}{m_{\tilde{l}_\alpha}^2}$$

$$\times\left[(A^{i\alpha a*}A^{i\alpha a} + B^{i\alpha a*}B^{i\alpha a})F_1(k_{\alpha a})\right.$$

$$\left.+\frac{m_{\chi_a^0}}{m_\mu}\mathrm{Re}(A^{i\alpha a*}B^{i\alpha a})F_2(k_{\alpha a})\right] \qquad (7)$$

and

$$\delta a_\mu^{(c)} = \frac{g_2^2}{16\pi^2}\frac{m_\mu^2}{m_{\tilde{\nu}_\alpha}^2}Z_{\tilde{\nu}}^{i\alpha}Z_{\tilde{\nu}}^{i\alpha*}$$
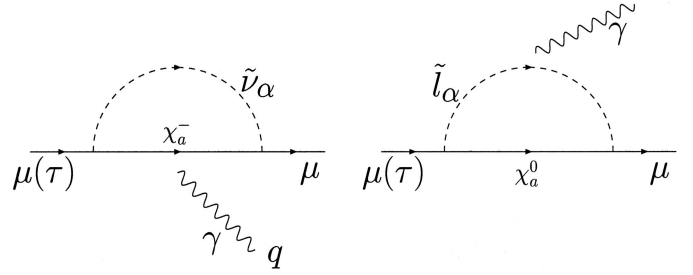

Fig. 1. Feynman diagrams of the one-loop SUSY contribution to a_μ (and the process $\tau \to \mu\gamma$) via the exchange of a chargino (left) and via a neutralino (right)

$$\times\left[\left(Z_{1a}^{+*}Z_{1a}^+ + \frac{m_\mu^2}{2M_W^2\cos^2\beta}Z_{2a}^{-*}Z_{2a}^-\right)F_3(k_{\alpha a})\right.$$

$$\left.+\frac{m_{\chi_a^-}}{\sqrt{2}M_W\cos\beta}\mathrm{Re}(Z_{1a}^+Z_{2a}^-)F_4(k_{\alpha a})\right], \qquad (8)$$

respectively with index $i = 2$, and $k_{\alpha a} = m_{\chi_a^0}^2/m_{\tilde{l}_\alpha}^2$ $(k_{\alpha a} = m_{\chi_a^-}^2/m_{\tilde{\nu}_\alpha}^2)$ for the neutralino (chargino) exchange. In the above expressions the A and B are the lepton–slepton–neutralino coupling vertices given by

$$A^{i\alpha a} = \left(Z_{\tilde{L}}^{i\alpha}(Z_N^{1a} + Z_N^{2a}\cot\theta_{\text{W}})\right.$$

$$\left.- \cot\theta_{\text{W}}\frac{m_i}{M_W\cos\beta}Z_{\tilde{L}}^{(i+3)\alpha}Z_N^{3a}\right), \qquad (9)$$

$$B^{i\alpha a} = -\left(2Z_{\tilde{L}}^{(i+3)\alpha}Z_N^{1a*} + \cot\theta_{\text{W}}\frac{m_i}{M_W\cos\beta}Z_{\tilde{L}}^{i\alpha}Z_N^{3a*}\right), \qquad (10)$$

where $Z_{\tilde{L}}$ is the 6×6 slepton mixing matrix and Z_N is the neutralino mixing matrix. Similarly, $Z_{\tilde{\nu}}$ is the sneutrino mixing matrix, while Z^+ and Z^- are the mixing matrices for the charginos. The definitions of these mixing matrices and the expressions of F_i's are given in the appendix.

For the processes $l_i \to l_j\gamma$ [10], the contribution from neutralino exchange gives

$$A_{\text{L}}^{ij(n)} = -\frac{1}{32\pi^2}\left(\frac{e}{\sqrt{2}\cos\theta_{\text{W}}}\right)^2\frac{1}{m_{\tilde{l}_\alpha}^2} \qquad (11)$$

$$\times\left[B^{j\alpha a*}B^{i\alpha a}F_1(k_{\alpha a}) + \frac{m_{\chi_a^0}}{m_i}B^{j\alpha a*}A^{i\alpha a}F_2(k_{\alpha a})\right],$$

$$A_{\text{R}}^{ij(n)} = A_{\text{L}}^{(n)}\ (B \leftrightarrow A), \qquad (12)$$

while the corresponding contribution coming from chargino exchange is

$$A_{\text{L}}^{ij(c)} = \frac{g_2^2}{32\pi^2}Z_{\tilde{\nu}}^{i\alpha*}Z_{\tilde{\nu}}^{j\alpha}\frac{1}{m_{\tilde{\nu}_\alpha}^2}\left[Z_{2a}^-Z_{2a}^{-*}\frac{m_im_j}{2M_W^2\cos^2\beta}F_3(k_{\alpha a})\right.$$

$$\left.+\frac{m_{\chi_a^-}}{\sqrt{2}M_W\cos\beta}Z_{1a}^+Z_{2a}^-\frac{m_j}{m_i}F_4(k_{\alpha a})\right], \qquad (13)$$

$$A_{\mathrm{R}}^{ij(c)} = \frac{g_2^2}{32\pi^2} Z_{\tilde{\nu}}^{i\alpha *} Z_{\tilde{\nu}}^{j\alpha} \frac{1}{m_{\tilde{\nu}_\alpha}^2} \tag{14}$$

$$\times \left[Z_{1a}^+ Z_{1a}^{+ *} F_3(k_{\alpha a}) + \frac{m_{\chi_a^-}}{\sqrt{2} M_W \cos\beta} Z_{1a}^{+ *} Z_{2a}^{- *} F_4(k_{\alpha a}) \right] .$$

2.2 Flavor structure in the interaction basis

The expressions for δa_μ and $\mathrm{Br}(l_i \to l_j\gamma)$ in the last subsection show that there are close relations between the two quantities. We notice that all these expressions are given in the mass eigenstates of the SUSY particles and the lepton flavor mixing is presented in the mixing matrices on the interaction vertices. These expressions are suitable for numerical calculations. However, to analyze the flavor structure of the amplitude, it is more convenient to work in the *interaction basis*, which is defined as the basis where the lepton mass matrix and the gauge coupling vertices are all diagonal. In this basis there are much more Feynman diagrams than those in Fig. 1. For example, the vertex A given in (9) actually contains three different interaction vertices, l_i^{L}–$\tilde{l}_i^{\mathrm{L}}$–$\tilde{B}$, l_i^{L}–$\tilde{l}_i^{\mathrm{L}}$–$\tilde{W}$, and l_i^{L}–$\tilde{l}_i^{\mathrm{R}}$–$\tilde{H}_D$ in this basis. Thus only the A^*A term in (7) represents 9 different Feynman diagrams.

In this basis the slepton and sneutrino mass matrices are generally not diagonal. We first give the form of these mass matrices. The slepton mass matrix can be written in a general form as follows:

$$M_{\tilde{l}}^2 = \begin{pmatrix} Z_{\mathrm{L}} m_{\mathrm{L}}^2 Z_{\mathrm{L}}^\dagger & -m_l(\mu\tan\beta + A_l^*) \\ -m_l(\mu^*\tan\beta + A_l) & Z_{\mathrm{R}} m_{\mathrm{R}}^2 Z_{\mathrm{R}}^\dagger \end{pmatrix} , \tag{15}$$

where

$$m_{\mathrm{L}}^2 = m_{\tilde{l}}^2 + m_l^2 + \cos 2\beta \left(-\frac{1}{2} + \sin^2\theta_{\mathrm{W}} \right) M_Z^2 , \tag{16}$$

$$m_{\mathrm{R}}^2 = m_{\tilde{r}}^2 + m_l^2 - \cos 2\beta \sin^2\theta_{\mathrm{W}} M_Z^2 , \tag{17}$$

and m_l is the diagonal mass matrix of leptons. Here $m_{\tilde{l}}^2$ and $m_{\tilde{r}}^2$ are diagonal matrices with their diagonal elements representing the mass squares of $(\tilde{e}_{\mathrm{L}}, \tilde{\mu}_{\mathrm{L}}, \tilde{\tau}_{\mathrm{L}})$ and $(\tilde{e}_{\mathrm{R}}, \tilde{\mu}_{\mathrm{R}}, \tilde{\tau}_{\mathrm{R}})$ respectively. Z_{L} and Z_{R} represent the mixing matrices in the left- and right-handed sleptons. In this work we consider the mixing between the second and the third generations (thereafter we will ignore the first generation). Z_{L} is then given by

$$Z_{\mathrm{L}} = \begin{pmatrix} c_{\mathrm{L}} & s_{\mathrm{L}} \\ -s_{\mathrm{L}} & c_{\mathrm{L}} \end{pmatrix} , \quad \text{with} \quad c_{\mathrm{L}} = \cos\theta_{\mathrm{L}}, \quad s_{\mathrm{L}} = \sin\theta_{\mathrm{L}} , \tag{18}$$

while m_{L}^2 is given by

$$m_{\mathrm{L}}^2 = \begin{pmatrix} m_2^2 & \\ & m_3^2 \end{pmatrix} . \tag{19}$$

In this basis the sneutrino mass matrix can be written as

$$M_{\tilde{\nu}}^2 = Z_{\mathrm{L}} m_{\tilde{l}}^2 Z_{\mathrm{L}}^\dagger + \frac{1}{2} \cos 2\beta M_Z^2 . \tag{20}$$

Before giving the relations between δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ in the next section, we first give an analysis of the flavor structure of the form factors A_{L} and A_{R} in the following. From (11)–(13), we can see that the flavor structure of $A_{\mathrm{L(R)}}$ is approximately proportional to $(M_{\tilde{\nu}}^2)_{ij}^{-1}$ or $(M_{\tilde{l}}^2)_{ij}^{-1}$, noticing that the functions $F_i(k_{\alpha a})$ are quite flat in an appropriate range of $k_{\alpha a}$. The numerical results in the next section justify our analysis given here[1]. It is easy to get, in the interaction basis,

$$(M_{\tilde{\nu}}^2)^{-1} = \begin{pmatrix} \frac{c_{\mathrm{L}}^2}{m_2^2} + \frac{s_{\mathrm{L}}^2}{m_3^2} & c_{\mathrm{L}} s_{\mathrm{L}} \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} \\ c_{\mathrm{L}} s_{\mathrm{L}} \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} & \frac{s_{\mathrm{L}}^2}{m_2^2} + \frac{c_{\mathrm{L}}^2}{m_3^2} \end{pmatrix} . \tag{21}$$

δa_μ is approximately proportional to the inverse mass square of $\tilde{\nu}_\mu - \tilde{\nu}_\mu$, which is denoted

$$F(\tilde{\nu}_\mu - \tilde{\nu}_\mu) = \frac{c_{\mathrm{L}}^2}{m_2^2} + \frac{s_{\mathrm{L}}^2}{m_3^2} , \tag{22}$$

while $\tau \to \mu\gamma$ is approximately proportional to that of $\tilde{\nu}_\tau - \tilde{\nu}_\mu$, which is

$$F(\tilde{\nu}_\tau - \tilde{\nu}_\mu) = \frac{1}{2} \sin 2\theta_{\mathrm{L}} \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} . \tag{23}$$

We consider the following two limit cases:

$$F(\tilde{\nu}_\mu - \tilde{\nu}_\mu) \to \begin{cases} \frac{1}{m^2}, & \text{if } m_2^2 \approx m_3^2 \approx m^2 , \\ \frac{s_{\mathrm{L}}^2}{m_3^2}, & \text{if } m_2^2 \gg m_3^2 , \end{cases} \tag{24}$$

while

$$F(\tilde{\nu}_\tau - \tilde{\nu}_\mu) \to \begin{cases} \frac{1}{2} \sin 2\theta_{\mathrm{L}} \frac{\Delta m^2}{m^4}, & \text{if } m_2^2 \approx m_3^2 \approx m^2 , \\ \frac{1}{2} \sin 2\theta_{\mathrm{L}} \frac{1}{m_3^2}, & \text{if } m_2^2 \gg m_3^2 . \end{cases} \tag{25}$$

In the first case with $m_2^2 \approx m_3^2 = m^2$, we can see that δa_μ does not depend on the mixing angle θ_{L} and has no direct relation with $\mathrm{Br}(\tau \to \mu\gamma)$. Thus models with gravity or gauge mediated supersymmetry breaking may predict that δa_μ has nothing to do with the mixing angle θ_{L}, as already noticed in [11]. Thus, to study δa_μ in the first case is actually equivalent to the case of no lepton flavor mixing in the soft sector, which has been extensively studied in the literature [5]. The second case leads us to the effective SUSY scenario [7], where the first two generations' sfermions are as heavy as about 20 TeV while the third generation's sfermions are kept in a few hundred GeV. In this case δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ are closely related; increasing θ_{L} to enhance δa_μ will unavoidably lead to large $\mathrm{Br}(\tau \to \mu\gamma)$. We have to consider the two quantities simultaneously and take the experimental bound on $\mathrm{Br}(\tau \to \mu\gamma)$ into account.

---

[1] In other works, such as in [6], similar relations are given under the approximation that all the SUSY particles are degenerate.

The inverse of the mass square of sleptons, in the interaction basis, is approximately given by

$$(M_{\tilde{l}}^2)^{-1} \approx \begin{pmatrix} A & C \\ C^\dagger & B \end{pmatrix} , \qquad (26)$$

with

$$A \approx (M_{\tilde{\nu}}^2)^{-1}, \quad B \approx A(\theta_L \to \theta_R) \qquad (27)$$

and

$$C \approx m_\tau \mu \tan\beta \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} \qquad (28)$$

$$\times \begin{bmatrix} \frac{1}{4}\sin 2\theta_L \sin 2\theta_R \frac{m_2^2-m_3^2}{m_2^2 m_3^2} & \frac{1}{2}\sin 2\theta_L \left(\frac{s_R^2}{m_2^2} + \frac{c_R^2}{m_3^2}\right) \\ \frac{1}{2}\sin 2\theta_R \left(\frac{s_L^2}{m_2^2} + \frac{c_L^2}{m_3^2}\right) & \left(\frac{s_L^2}{m_2^2} + \frac{c_L^2}{m_3^2}\right)\left(\frac{s_R^2}{m_2^2} + \frac{c_R^2}{m_3^2}\right) \end{bmatrix} .$$

In the matrix C we have omitted the terms proportional to m_μ. From the above expressions we know that $F(\tilde{\mu}_L - \tilde{\mu}_L)$ is the same as that of $F(\tilde{\nu}_\mu - \tilde{\nu}_\mu)$, while $F(\tilde{\mu}_R - \tilde{\mu}_R)$ is gotten by changing θ_L to θ_R in $F(\tilde{\nu}_\mu - \tilde{\nu}_\mu)$. The most interesting result is that of $F(\tilde{\mu}_L - \tilde{\mu}_R)$, given by

$$F(\tilde{\mu}_L - \tilde{\mu}_R) = \frac{1}{4}m_\tau \mu \tan\beta \sin 2\theta_L \sin 2\theta_R \left(\frac{m_2^2 - m_3^2}{m_2^2 m_3^2}\right)^2 . \qquad (29)$$

The other two quantities related to $\tau \to \mu\gamma$ are

$$F(\tilde{\mu}_L - \tilde{\tau}_R) \qquad (30)$$
$$= \frac{1}{2}m_\tau \mu \tan\beta \sin 2\theta_L \left(\frac{m_2^2 - m_3^2}{m_2^2 m_3^2}\right)\left(\frac{s_R^2}{m_2^2} + \frac{c_R^2}{m_3^2}\right) ,$$

and

$$F(\tilde{\mu}_R - \tilde{\tau}_L) \qquad (31)$$
$$= \frac{1}{2}m_\tau \mu \tan\beta \sin 2\theta_R \left(\frac{m_2^2 - m_3^2}{m_2^2 m_3^2}\right)\left(\frac{s_L^2}{m_2^2} + \frac{c_L^2}{m_3^2}\right) ,$$

respectively.

Similarly, we consider the limit case of nearly degenerate sleptons, $m_2^2 \approx m_3^2 \approx m^2$. The other term, omitted in (29), which is proportional to m_μ, may become important. Then we have

$$F(\tilde{\mu}_L - \tilde{\mu}_R)$$
$$\approx \frac{\mu \tan\beta}{m^4}\left[m_\mu + \frac{1}{4}m_\tau \sin 2\theta_L \sin 2\theta_R \left(\frac{\Delta m^2}{m^2}\right)^2\right]$$
$$\approx \frac{\mu \tan\beta}{m^4}m_\mu . \qquad (32)$$

We can thus reach the same conclusion as before, that is, if $m_2^2 \approx m_3^2 \approx m^2$, δa_μ has no direct relation to the slepton mixing angles. In case of $m_2 \gg m_3$, we have

$$F(\tilde{\mu}_L - \tilde{\mu}_R) \approx \frac{\mu \tan\beta}{m_3^4}\left[m_\mu s_L^2 s_R^2 + \frac{1}{4}m_\tau \sin 2\theta_L \sin\theta_R\right] , \qquad (33)$$

where the second term may dominate. In this case δa_μ depends crucially on the mixing angles θ_L and θ_R.

The key feature of the effective Lagrangian in (4) is that there is a chiral flip between the initial and final fermion states. This feature leads to the circumstance that, in the case of no slepton mixing, all terms in (7) and (8) will produce at least one muon mass, m_μ, suppression, which either comes from the mass insertion on the external fermion legs, or from the Yukawa coupling vertices, or from the left- and right-handed smuon mixing. This can be explicitly examined by checking all the Feynman diagrams in the interaction basis. The quite interesting point in the case of slepton mixing is that $F(\tilde{\mu}_L - \tilde{\mu}_R)$, given in (29), is approximately proportional to m_τ, which can give an enhancement to δa_μ. This term may dominate the other ones if both the left- and right-handed mixing angles are large. We will show this point in the next section.

From the above analysis we have shown that by changing to the interaction basis, δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ can manifest their dependence on the SUSY and mixing parameters. This basis becomes very convenient for our discussion of the relation between δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ later.

It should be mentioned that the parameters $m_{2,3}$ are different for sleptons and sneutrinos, as shown in (15)–(17) and (20). We adopt the same symbol in $F(\tilde{\nu} - \tilde{\nu})$ and $F(\tilde{\mu} - \tilde{\mu})$ only for simplicity. In numerical calculations we adopt the full form in (15) and (20).

3 Bound on δa_μ and numerical results

In this section, we focus our discussion on the effective SUSY scenario, i.e., $m_2 \gg m_3$. In this case, δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ are closely related, as shown in the last section. We will give approximate bounds on δa_μ through analytic relations between δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$, taking into account the experimental up limit on the LFV processes. Numerical results are also presented.

The free parameters in this calculation are the Higgsino mass, μ, the $U(1)_Y$ and $SU(2)_W$ gaugino masses, M_1 and M_2, the ratio of VEVs, $\tan\beta$, the mixing angles, θ_L and θ_R, the slepton mass squares, $m_{\tilde{l}_2}^2, m_{\tilde{l}_3}^2, m_{\tilde{r}_2}^2, m_{\tilde{r}_3}^2$, and the trilinear terms A_l. Throughout the whole calculation we fix $A_l = 0$, $m_{\tilde{l}_2}^2 = m_{\tilde{r}_2}^2 = 20\,\mathrm{TeV}$. If we do not explicitly state differently, we will take the relations $M_1 = \frac{5\alpha_1}{3\alpha_2}M_2$ and $m_{\tilde{l}_3}^2 = m_{\tilde{r}_3}^2$ to hold. We demand all the SUSY particle spectra to be above the present experimental lower limit.

3.1 δa_μ with $\theta_R = 0$

When there is only left-handed mixing in the slepton sector, the most important contribution to δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ comes from the diagram in Fig. 2, given in the interaction basis. From this diagram we can directly read that

$$A_R^{23}(c) \approx \frac{1}{2}\delta a_\mu^{(c)}/m_\mu^2 \frac{F(\tilde{\nu}_\tau - \tilde{\nu}_\mu)}{F(\tilde{\nu}_\mu - \tilde{\nu}_\mu)} \approx \frac{1}{2}\delta a_\mu^{(c)}/m_\mu^2 \frac{c_L}{s_L} . \qquad (34)$$
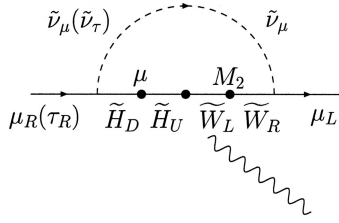
Fig. 2. Feynman diagram which gives the dominant contribution to δa_μ (and to the process $\tau \to \mu\gamma$) in the case of only left-handed slepton mixing. The black dots in the chargino line are mass insertions, with the middle dot representing $\sqrt{2}M_W \sin\beta$
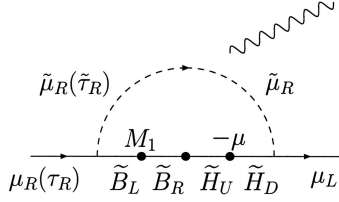

Fig. 3. Feynman diagram which gives the dominant contribution to δa_μ (and to the process $\tau \to \mu\gamma$) in the case of only right-handed slepton mixing. The black dots in the neutralino line are mass insertions, with the middle dot representing $M_Z \sin\beta \sin\theta_W$

Then we have, assuming $\theta_L = \pi/4$,

$$\mathrm{Br}(\tau \to \mu\gamma) \approx \frac{\alpha_{\mathrm{em}}}{4} m_\tau^5 |A_R^{23}(c)|^2 / \Gamma_\tau$$

$$\approx \frac{\alpha_{\mathrm{em}}}{4} m_\tau^5 / \Gamma_\tau \left| \frac{\delta a_\mu^{(c)}}{2m_\mu^2} \frac{c_L}{s_L} \right|^2$$

$$\approx 2.9 \times 10^{13} |\delta a_\mu|^2 . \tag{35}$$

From the present experimental upper bound on $\mathrm{Br}(\tau \to \mu\gamma)$ ($< 10^{-6}$ [12]), we get

$$\delta a_\mu < 1.9 \times 10^{-10}, \quad \text{in case of } \theta_R = 0. \tag{36}$$

From this diagram we also have the conclusion that

$$\mu M_2 > 0, \quad \text{in case of } \theta_R = 0 \tag{37}$$

in order that SUSY gives a positive contribution to δa_μ. The same diagram gives the dominant contribution to δa_μ in the case of no lepton flavor mixing. Thus the same conclusion of the sign of μ is given in that case.

A numerical study verifies our above estimation.

3.2 δa_μ with $\theta_L = 0$

In the case of only right-handed mixing, the chargino-sneutrino diagram gives no contribution to δa_μ. The most important contribution to δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ comes from the diagram in Fig. 3, given in the interaction basis. If we ignore the mixing between the left- and right-handed sleptons, Z_R is approximately the slepton mixing matrix.

From Fig. 3 we have

$$A_R^{23}(n) \approx \frac{1}{2} \frac{\delta a_\mu^{(n)}}{m_\mu^2} \left(\frac{m_\mu}{m_\tau} \right) \frac{F(\tilde\tau_R - \tilde\mu_R)}{F(\tilde\mu_R - \tilde\mu_R)}$$

$$\approx \frac{1}{2} \frac{\delta a_\mu^{(n)}}{m_\mu^2} \left(\frac{m_\mu}{m_\tau} \right) \frac{c_R}{s_R} . \tag{38}$$

Then we have, assuming $\theta_R = \pi/4$,

$$\mathrm{Br}(\tau \to \mu\gamma) \approx \frac{\alpha_{\mathrm{em}}}{4} m_\tau^5 \left| A_R^{23}(n) \right|^2 / \Gamma_\tau$$

$$\approx \frac{\alpha_{\mathrm{em}}}{4} m_\tau^5 / \Gamma_\tau \left| \frac{\delta a_\mu^{(n)}}{2m_\mu^2} \frac{m_\mu}{m_\tau} \frac{c_R}{s_R} \right|^2$$

$$\approx 1. \times 10^{11} |\delta a_\mu|^2 . \tag{39}$$

From the present upper limit of $\mathrm{Br}(\tau \to \mu\gamma) < 10^{-6}$, we get

$$\delta a_\mu < 32 \times 10^{-10}, \quad \text{in the case of } \theta_L = 0. \tag{40}$$

This upper bound is much larger than that in the case of only left-handed mixing. It is obvious that the factor $\frac{m_\mu}{m_\tau}$ in (38), which greatly suppresses $\mathrm{Br}(\tau \to \mu\gamma)$, helps to increase the bound. This factor comes from the μ_L–$\widetilde{H}_D$–$\tilde\mu_R$ Yukawa coupling vertex in Fig. 3, where the Higgsino component $\widetilde{H}_D$ has to be associated with the muon line since there is only right-handed mixing in the slepton sector. However, in Fig. 2, where the charged Higgsino component $\widetilde{H}_D$ is associated with the tau line, no such factor helps to suppress $\mathrm{Br}(\tau \to \mu\gamma)$.

Another interesting point is that the mass insertion for the neutral component of $\widetilde{H}_U \widetilde{H}_D$ is $-\mu$, while it is μ for the same term of the charged component. These terms are clearly shown in the mass matrices of charginos and neutralinos in the appendix. This sign difference comes in when we contract the SUSY invariant term $\mu\epsilon_{ab} H_D^a H_U^b$. Thus we have

$$\mu M_1 < 0, \quad \text{in the case of } \theta_L = 0, \tag{41}$$

to give a positive contribution to δa_μ. This means that if we set M_1 and M_2 to have the same sign, which is well motivated theoretically, μ should be negative in this case. We have numerically demonstrated this point by changing the signs of M_1 and μ simultaneously and finding that δa_μ almost has the same value.

Since we ignored the left–right mixing between the sleptons, the naive bound we get in (40) should be examined numerically. The numerical results in this case are shown in Fig. 4. In this figure (and all similar figures below) we draw δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ in the same figure as functions of $m_{\tilde l_3} = m_{\tilde r_3} = m_3$. The upper group of curves represent $\mathrm{Br}(\tau \to \mu\gamma)$ while the corresponding curve in the lower group is δa_μ with the same parameters. The two solid horizontal lines represent the E821 $\pm 2.5\sigma$ bounds, $\delta a_\mu = 6.5, \ 61.5 \times 10^{-10}$. The dotted horizontal
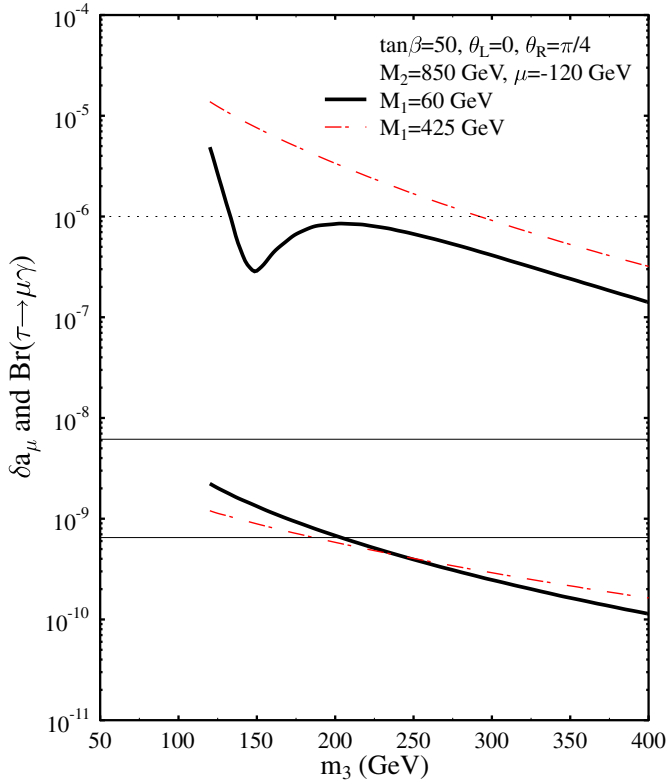
Fig. 4. δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ as functions of $m_3 = m_{\tilde{l}_3} = m_{\tilde{r}_3}$ in the case of $\theta_L = 0$, $\theta_R = \pi/4$. The other parameters are $\tan\beta = 50$, $M_2 = 850\,\mathrm{GeV}$. M_1 is fixed to be $60\,\mathrm{GeV}$ for the a solid line and $0.5M_2$ for the dashed line. The horizontal lines represent the E821 $\pm 2.5\sigma$ bounds (solid) and the upper limit of $\mathrm{Br}(\tau \to \mu\gamma)$ (dotted)
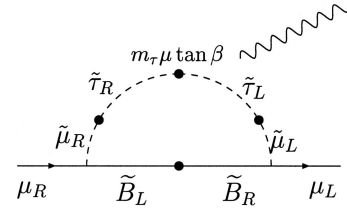

Fig. 5. Feynman diagram which gives the dominant contribution to δa_μ in the case that both the left- and right-handed slepton mixing are large
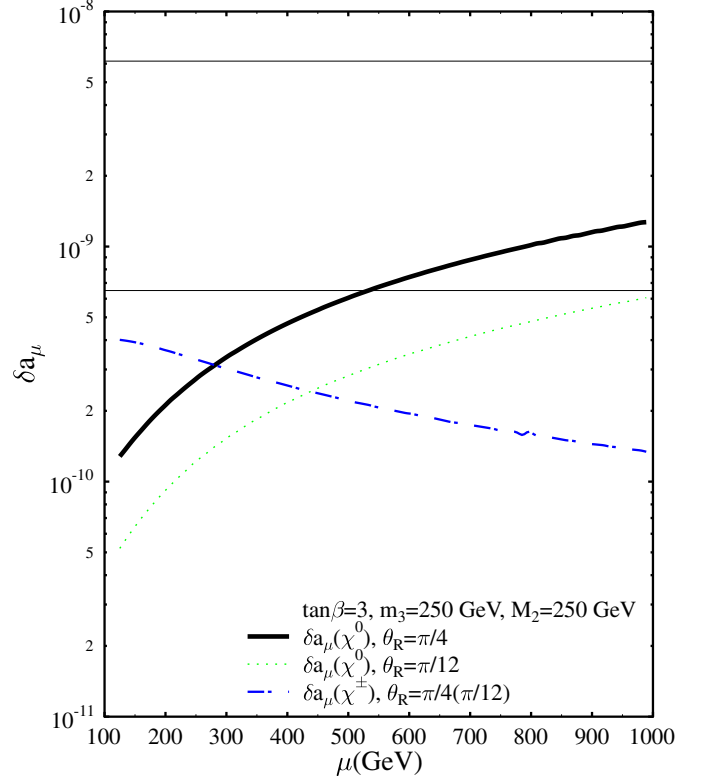

Fig. 6. δa_μ as function of μ for $\tan\beta = 3$, $M_2 = 250\,\mathrm{GeV}$, $m_3 = 250\,\mathrm{GeV}$ and $\theta_L = \pi/4$. $\delta a_\mu(\chi^0)$ and $\delta a_\mu(\chi^\pm)$ represent the contribution coming from exchanging neutralino and chargino respectively. The horizontal lines represent the E821 $\pm 2.5\sigma$ bounds of δa_μ

line is the experimental upper bound on the branching ratio of $\tau \to \mu\gamma$, $\mathrm{Br}(\tau \to \mu\gamma) = 10^{-6}$.

We take a large $\tan\beta$ ($= 50$) and M_2 ($= 850\,\mathrm{GeV}$), while μ is negative. If we adopt the GUT motivated relation $M_1 = \frac{5\alpha_1}{3\alpha_2}M_2 \approx 0.5M_2$ we have $\delta a_\mu < 3 \times 10^{-10}$ to satisfy the $\mathrm{Br}(\tau \to \mu\gamma)$ bound. However, if we relax the above relation and fix $M_1 = 60\,\mathrm{GeV}$, δa_μ can be as large as $\sim 17 \times 10^{-10}$ without violating the bound of $\mathrm{Br}(\tau \to \mu\gamma)$. This case corresponds to the LSP (lightest supersymmetric particle) being a bino, which is much lighter than the other neutralinos.

3.3 δa_μ with $\theta_L \neq 0$ and $\theta_R \neq 0$

This case is the most general and most interesting one. Our numerical calculation mainly focuses on this case. In this case we have derived in (29) that there is an m_τ enhancement to $F(\tilde{\mu}_L - \tilde{\mu}_R)$ if both the left- and right-handed mixing is large in the slepton sector. The enhancement leads to the result that the diagram in Fig. 5 may give a dominant contribution to δa_μ if both θ_L and θ_R are large. However, there is no obvious term which gives a dominant contribution to $\mathrm{Br}(\tau \to \mu\gamma)$. We find that in the small m_3 region the diagram in Fig. 5 with μ_R replaced by τ_R may dominate other terms contributing to $\mathrm{Br}(\tau \to \mu\gamma)$. In this

case we get a similar limit as that given in the case with only right-handed mixing, i.e.,

$$\delta a_\mu < 32 \times 10^{-10} \text{ in the case of } \theta_L \neq 0 \text{ and } \theta_R \neq 0 . \quad (42)$$

However, this bound is very loose because in a large part of parameter space the contribution to $\mathrm{Br}(\tau \to \mu\gamma)$ by exchanging $\chi^\pm$ is more important than that by exchanging χ^0. We cannot get a simple relation between δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ for this case and we have to study it numerically.

At first we will numerically verify that Fig. 5 indeed gives an important contribution to δa_μ by displaying the contributions from exchanging χ^0 and $\chi^\pm$ separately in Fig. 6. From this figure, we notice that the contribution from exchanging a neutralino depends strongly on the val-
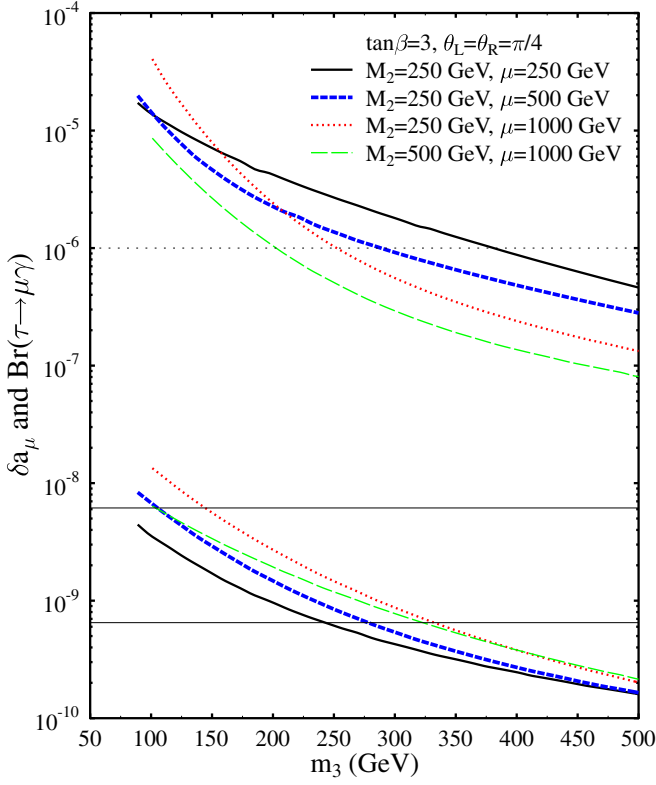
Fig. 7. δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ as functions of m_3 for $\theta_L = \pi/4$, $\theta_R = \pi/4$, $\tan\beta = 3$, $M_2 = 250, 500\,\mathrm{GeV}$ and $\mu = 250, 500, 1000\,\mathrm{GeV}$. The horizontal lines represent the E821 $\pm 2.5\sigma$ bounds of δa_μ (solid) and the upper limit of $\mathrm{Br}(\tau \to \mu\gamma)$ (dotted)
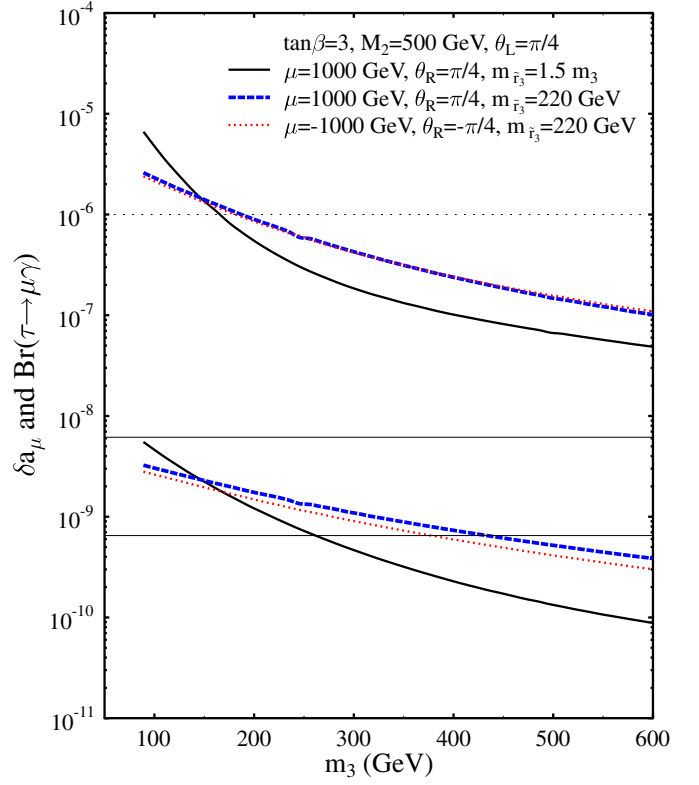
Fig. 8. δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ as functions of $m_{\tilde{l}_3} = m_3$ for $\tan\beta = 3$, $M_2 = 500\,\mathrm{GeV}$ and $\theta_L = \pi/4$. Taking $m_{\tilde{r}_3} = 1.5 m_3$, $220\,\mathrm{GeV}$, $\mu = \pm 1000\,\mathrm{GeV}$ and $\theta_R = \pm\pi/4$ respectively. The horizontal lines represent the E821 $\pm 2.5\sigma$ bounds of δa_μ (solid) and the upper limit of $\mathrm{Br}(\tau \to \mu\gamma)$ (dotted)

ues of θ_R, while that from exchanging charginos has no change by changing the values of θ_R. From (29) it is obvious that the neutralino contribution becomes large as μ increases, while the chargino contribution becomes small since a large μ leads to a heavy chargino mass.

Figure 7 displays δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ for $\tan\beta = 3$, $\theta_L = \theta_R = \pi/4$. If both M_2 and μ are large, there is a large region which can accommodate δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ simultaneously. As μ becomes large, $\mathrm{Br}(\tau \to \mu\gamma)$ decreases while δa_μ increases. This is understood as follows: a large μ enhances $F(\tilde{\mu}_L - \tilde{\mu}_R)$ and leads to a large chargino mass, which decreases $\mathrm{Br}(\tau \to \mu\gamma)$.

In Fig. 8 we relax the relation $m_{\tilde{l}_3} = m_{\tilde{r}_3}$. We show δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ as functions of $m_{\tilde{l}_3} = m_3$ for $m_{\tilde{r}_3} = 1.5 m_3$ and $m_{\tilde{r}_3} = 220\,\mathrm{GeV}$. From (29) we notice that the sign of μ can be either positive or negative, depending on the relative sign of θ_L and θ_R. We also plot δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ on changing the sign of μ and θ_R simultaneously in Fig. 8. There is little effect on δa_μ by the sign reverse.

Since $\mathrm{Br}(\tau \to \mu\gamma)$ is approximately proportional to $\tan^2\beta$, its upper limit constrains $\tan\beta$ strongly. In the above figures we take $\tan\beta = 3$. In Fig. 9 we plot δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ as functions of $m_{\tilde{l}_3} = m_3$ for $\tan\beta = 10$, and $\mu = 1000\,\mathrm{GeV}$, $M_2 = 400, 800\,\mathrm{GeV}$, $M_1 = \frac{5\alpha_1}{3\alpha_2} \approx 0.5 M_2$, $60\,\mathrm{GeV}$ respectively. We fix $m_{\tilde{r}_3} = 300\,\mathrm{GeV}$ in

this figure. We can see that when $\tan\beta$ is as large as 10 there is still a large region for m_3 to accommodate δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ simultaneously if M_2 is large. Generally m_3 takes a larger value than in the case of $\tan\beta = 3$ to satisfy the δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ bounds. We also notice that relaxing the relation $M_1 = 0.5 M_2$ does not change the result much.

In summary, in the effective SUSY scenario, when both the left- and right-handed slepton mixing is large, SUSY can enhance δa_μ to within the E821 $\pm 2.5\sigma$ bounds in a large parameter space, which can satisfy the constraints by the experimental limit on $\mathrm{Br}(\tau \to \mu\gamma)$. In this case a small $\tan\beta$ is slightly favored. The Higgsino mass parameter μ can be either positive or negative depending on the relative sign between θ_L and θ_R. We find that δa_μ can reach up to $\sim 20 \times 10^{-10}$ even keeping the relation $M_1 \approx 0.5 M_2$, implying that the bino is not necessarily kept very light in this case.

4 Summary and conclusions

In this work we study the correlation between the SUSY contribution to δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ by translating the analytic expressions in the mass eigenstates to the interaction basis, where the gauge coupling and Yukawa interaction vertices are all diagonal. If the slepton mass eigen-
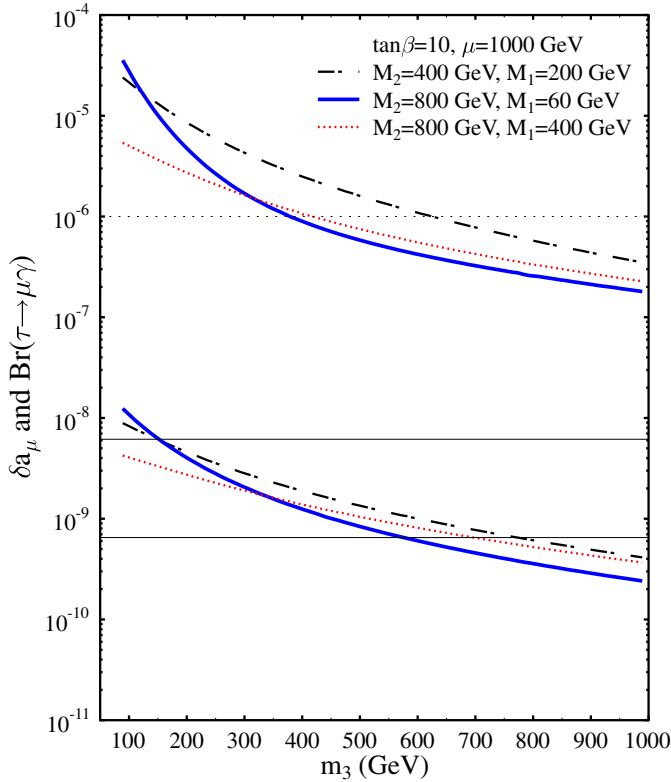
Fig. 9. δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ as functions of $m_{\tilde{l}_3} = m_3$ for $\tan\beta = 10$, $\mu = 1000\,\mathrm{GeV}$, $\theta_\mathrm{L} = \theta_\mathrm{R} = \pi/4$, $m_{\tilde{r}_3} = 300\,\mathrm{GeV}$, $M_2 = 400, 800\,\mathrm{GeV}$ and $M_1 = 0.5 M_2, 60\,\mathrm{GeV}$. The horizontal lines represent the E821 $\pm 2.5\sigma$ bounds of δa_μ (solid) and the upper limit of $\mathrm{Br}(\tau \to \mu\gamma)$ (dotted)

states are approximately degenerate, δa_μ does not depend on the lepton flavor mixing angles and has no direct relation with the LFV processes. In this case, the analysis of δa_μ is actually the same as no slepton mixing, as most authors have done in the literature. Another case, that the second (and first) generation slepton is much heavier than the third generation slepton, corresponds to the scenario of effective SUSY. We mainly investigate this case in our work.

In the effective SUSY scenario, if there is only left-handed mixing on the slepton sector, the upper limit of $\mathrm{Br}(\tau \to \mu\gamma)$ constrains the SUSY contribution to a_μ to be smaller than 1.9×10^{-10}. In the case of only right-handed mixing in the slepton sector, a numerical study shows that δa_μ can be at most $\sim 17 \times 10^{-10}$ if a bino is the LSP and much lighter than other neutralinos. In this case the Higgsino mass μ is negative relative to M_1 in order to give a positive contribution to a_μ. In the case of both left- and right-handed mixing angles being large, we find the diagram exchanging bino can give a dominant contribution to δa_μ. The sign of μ is determined by making this diagram positive. Thus μ can be either positive or negative, depending on the relative sign between the left- and right-handed slepton mixing angles. A numerical study shows that in this case there is a large part of parameter space accommodating δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$ simultaneously. The

SUSY contribution to a_μ can reach up to $\sim 20 \times 10^{-10}$, without requiring a very light bino.

Our study shows that the parameter space is quite different in the effective SUSY scenario compared with that in the case of no slepton mixing. The small $\tan\beta$ value is more favored. The sign of μ is not constrained by the $(g_\mu - 2)$ experiment. Finally the effective SUSY scenario cannot be excluded by the E821 experiment if we take the lepton flavor mixing effects into account.

Appendix

In this appendix we present our conventions for the SUSY parameters and some analytic expressions for δa_μ and $\mathrm{Br}(\tau \to \mu\gamma)$. For the most part we adopt the conventions given in [13].

The two-component charged Higgsinos, $\widetilde{H}_D^2$, $\widetilde{H}_U^1$, and the charged winos, $\lambda^\pm = \frac{1}{\sqrt{2}}(\lambda_W^1 \mp i\lambda_W^2)$, combine to give two four-component Dirac fermions named charginos, where $\widetilde{H}_D^2$ and $\widetilde{H}_U^1$ are the second and the first components of the down and up Higgsino SU(2) doublets respectively, λ_W^1 and λ_W^2 are the first and the second components of the wino SU(2) triplet. The mass matrix of the charginos, given in the interaction eigenstates, is

$$M_\chi = \begin{bmatrix} m_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu \end{bmatrix} . \qquad (43)$$

The unitary mixing matrices Z^-, Z^+ satisfy

$$(Z^-)^\mathrm{T} M_\chi Z^+ = \mathrm{diag}\,(m_{\chi_1}, m_{\chi_2}) , \qquad (44)$$

which are defined by

$$\begin{pmatrix} -\mathrm{i}\,\lambda^- \\ \widetilde{H}_D^2 \end{pmatrix} = Z^- \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \end{pmatrix} , \qquad (45)$$

and

$$\begin{pmatrix} -\mathrm{i}\,\lambda^+ \\ \widetilde{H}_U^1 \end{pmatrix} = Z^+ \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} . \qquad (46)$$

The four-component Dirac charginos are defined by

$$\chi_i^+ = \begin{bmatrix} \varphi_i^+ \\ \overline{\varphi_i^-} \end{bmatrix} .$$

The mass term which will appear in the final form of Lagrangian is $-m_{\chi_i}\overline{\chi_i}\chi_i$.

The third component of the wino, λ_W^3, the bino, $\tilde{B}$, and the neutral Higgsinos, $\widetilde{H}_D^1$, $\widetilde{H}_U^2$, combine to give four Majorana neutralinos. The mass matrix for neutralinos in the interaction basis is given by (47)

$$M_{\chi^0} = \begin{bmatrix} m_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & m_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & 0 & -\mu \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & -\mu & 0 \end{bmatrix}, \tag{47}$$

which is diagonalized by

$$Z_N^{\mathrm{T}} M_{\chi^0} Z_N = \mathrm{diag}\left(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}\right). \tag{48}$$

The unitary mixing matrix Z_N is defined by

$$\begin{pmatrix} -\mathrm{i}\lambda_B \\ -\mathrm{i}\lambda_W^3 \\ \widetilde{H}_D^1 \\ \widetilde{H}_U^2 \end{pmatrix} = Z_N \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \\ \varphi_3^0 \\ \varphi_4^0 \end{pmatrix}. \tag{49}$$

The four-component Majorana neutralinos are given by

$$\chi_i^0 = \begin{bmatrix} \varphi_i^0 \\ \overline{\varphi_i^0} \end{bmatrix}.$$

The mass term in the final form of Lagrangian of neutralino is $-\frac{1}{2} m_{\chi_i^0} \overline{\chi_i^0} \chi_i^0$.

The mass matrices for the sleptons and sneutrinos have been given in (15) and (20). They are diagonalized by

$$Z_{\tilde{L}}^{\dagger} M_{\tilde{l}}^2 Z_{\tilde{L}} = \mathrm{diag}(m_{\tilde{l}_\alpha}^2), \ \ \alpha = 1, \cdots, 6, \tag{50}$$

and

$$Z_{\tilde{\nu}}^{\dagger} M_{\tilde{\nu}}^2 Z_{\tilde{\nu}} = \mathrm{diag}(m_{\tilde{\nu}_\alpha}^2), \ \ \alpha = 1, 2, 3. \tag{51}$$

The relation between the gauge eigenstates and the mass eigenstates are (omitting the first generation)

$$\begin{pmatrix} \tilde{\mu}_{\mathrm{L}} \\ \tilde{\tau}_{\mathrm{L}} \\ \tilde{\mu}_{\mathrm{R}}^* \\ \tilde{\tau}_{\mathrm{R}}^* \end{pmatrix} = Z_{\tilde{L}} \begin{pmatrix} m_{\tilde{l}_1}^2 \\ m_{\tilde{l}_2}^2 \\ m_{\tilde{l}_3}^2 \\ m_{\tilde{l}_4}^2 \end{pmatrix} \tag{52}$$

and similar expression for sneutrinos.

The functions $F_i(k)$ in (7) and (8) and (11)–(13) are given as follows. For neutralino exchange we have

$$F_1(k) = \frac{1 - 6k + 3k^2 + 2k^3 - 6k^2 \log k}{6(1-k)^4}, \tag{53}$$

$$F_2(k) = \frac{1 - k^2 + 2k \log k}{(1-k)^3}, \tag{54}$$

with $k_{\alpha a} = m_{\chi_a^0}^2 / m_{\tilde{l}_\alpha}^2$. For chargino exchange we have

$$F_3(k) = \frac{2 + 3k - 6k^2 + k^3 + 6k \log k}{6(1-k)^4}, \tag{55}$$

$$F_4(k) = \frac{3 - 4k + k^2 + 2 \log k}{(1-k)^3}, \tag{56}$$

with $k_{\alpha a} = m_{\chi_a^-}^2 / m_{\tilde{\nu}_\alpha}^2$.

References

1. G.W. Bennet et al., Muon g-2 Collaboration, Phys. Rev. Lett. **89**, 101804 (2002); erratum **89**, 129903 (2002), hep-ex/0208001
2. H.N. Brown et al., Muon g-2 Collaboration, Phys. Rev. Lett. **86**, 2227 (2001)
3. K. Hagiwara, A.D. Martin, Daisuke Nomura, T. Teubner, Phys. Lett. B **557**, 69 (2003), hep-ph/0209187
4. M. Davier, S. Eidelman, A. Hocker, Z. Zhang, Eur. Phys. J. C **27**, 497 (2003), hep-ph/0208177
5. L.L. Everett, G.L. Kane, S. Rigolin, L.T. Wang, Phys. Rev. Lett. **86**, 3484 (2001); J.L. Feng, K.T. Matchev, Phys. Rev. Lett. **86**, 3480 (2001); E.A. Baltz, P. Gondolo, Phys. Rev. Lett. **86**, 5004 (2001); U. Chattopadhyay, P. Nath, Phys. Rev. Lett. **86**, 5854 (2001); R. Arnowitt, B. Dutta, B. Hu, Y. Santoso, Phys. Lett. B **505**, 177 (2001); S. Komine, T. Moroi, M. Yamaguchi, Phys. Lett. B **506**, 93 (2001); J.R. Ellis, D.V. Nanopoulos, K.A. Olive, Phys. Lett. B **508**, 65 (2001); J. Hisano, K. Tobe, Phys. Lett. B **510**, 197 (2001); Z. Chacko, Graham D. Kribs, Phys. Rev. D **64**, 075015 (2001); K. Choi, K. Hwang, S.K. Kang, K.Y. Lee, W.Y. Song, Phys. Rev. D **64**, 055001 (2001); S.P. Martin, J.D. Wells, Phys. Rev. D **64**, 035003 (2001); S. Komine, T. Moroi, M. Yamaguchi, Phys. Lett. B **507**, 224 (2001); S. w. Baek, P. Ko, H.S. Lee, Phys. Rev. D **65**, 035004 (2002); D.F. Carvalho, J.R. Ellis, M.E. Gomez, S. Lola, Phys. Lett. B **515**, 323 (2001); H. Baer, C. Balazs, J. Ferrandis, X. Tata, Phys. Rev. D **64**, 035004 (2001); S. w. Baek, T. Goto, Y. Okada, K. i. Okumura, Phys. Rev. D **64**, 095001 (2001); G.C. Cho, K. Hagiwara, Phys. Lett. B **514**, 123 (2001); E.A. Baltz, P. Gondolo, astro-ph/0207673; U. Chattopadhyay, P. Nath, hep-ph/0208012; M. Byrne, C. Kolda, J.E. Lennon, hep-ph/0208067
6. S. Baek, P. Ko, J.H. Park, Eur. Phys. J. C **24**, 613 (2002), hep-ph/0203251; G.C. Cho, N. Haba, J. Hisano, Phys. Lett. B **529**, 117 (2002), hep-ph/0112163
7. M. Dine, A. Kagan, S. Samuel, Phys. Lett. B **243**, 250 (1990); S. Dimopoulos, G.F. Giudice, Phys. Lett. B **357**, 573 (1995); A. Pomarol, D. Tommasini, Nucl. Phys. B **466**, 3 (1996); A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Lett. B **388**, 588 (1996); D.E. Kaplan, F. Lepeintre, A. Masiero, A.E. Nelson, A. Riotto, Phys. Rev. D **60**, 055003

(1999); J. Hisano, K. Kurosawa, Y. Nomura, Phys. Lett.
B **445**, 316 (1999); J. Hisano, K. Kurosawa, Y. Nomura,
Nucl. Phys. B **584**, 3 (2000)

8. K. Hagiwara et al., Phys. Rev. D **66**, 01001 (2002)
9. Stephen P. Martin, James D. Wells, Phys. Rev. D **64**,
035003 (2001), and references therein

10. J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi, Phys. Rev.
D **53**, 2442 (1996)
11. X.J. Bi, Y.B. Dai, Phys. Rev. D **66**, 076006 (2002)
12. A. Abe et al., BELLE Collaboration, BELLE-CONF-0118
13. J. Rosiek, Phys. Rev. D **41**, 3464 (1990), hep-ph/9511250